Neutron Lifetimes and Void Coefficients for Research Reactors

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Prompt neutron lifetimes and void coefficients were calculated for research reactors using M.T.R.-type fuel elements, moderators of light and heavy water, and reflectors of light water, heavy water, graphite, and beryllium. Heavy-water-reflected and -moderated research reactors may have neutron lifetimes of the order of 0.001 sec. as compared with about 0.00006 sec. for a light-water-reflected and -moderated research reactor. Lifetime of the light-water core can be improved considerably by use of better reflectors, but at a substantial reduction and even reversal in sign of the void coefficient.

A desirable and important safety feature of a nuclear reactor is the automatic shutdown of the reactor by temperature rise and/or boiling in case of a sudden excursion in power level. To allow for fuel depletion, fission-product buildup, and possible consumption of neutrons by experiments, an operating reactor must have excess reactivity available from an amount of fuel greater than that required to make the reactor just critical in the cold, clean condition. The reactor is inherently safe if this available excess reactivity can be suddenly imposed on the reactor without melting of fuel elements or creation of destructive pressure surges before the imposed reactivity is canceled by effects of heat liberation.

A dramatic demonstration of the results of sudden addition of excess reactivity to a light-water research reactor of the swimming-pool type was given by the borax tests at Arco, Idaho (*1*). With sudden addition of a small amount of excess reactivity to the reactor operating at 80°F. the reactor shut itself off safely by boiling the water moderator and coolant. For those experiments which resulted in safe shutdown of the reactor operating initially at 80°F. the shortest initial period studied was "about 0.013 sec.," corresponding to the sudden introduction of 1.24% excess reactivity. Sudden addition of 3.3% excess reactivity resulted in a violent pressure surge which destroyed the reactor.

If the imposed excess reactivity is considerably greater than the delayed neutron fraction β, the period T of the power transient can be approximated by

$$T = \frac{l}{k_{eff}(1 - \beta) - 1} \qquad (1)$$

If the power surge boils the coolant and/or moderator, k_{eff} will change as boiling occurs:

$$k_{eff} = k^0_{eff} + \alpha V_v \qquad (2)$$

k^0_{eff} = effective reproduction factor at the beginning of the power surge

α = steam void coefficient, or the increase in k_{eff} per unit increase in steam volume. This is assumed to be constant for a given reactor.

V_v = volume of steam within the reactor core at time t

To minimize danger to a water-cooled and -moderated reactor in a power transient, it is desirable that the reactor period be large to allow time for heat to transfer from fuel elements into the water and to allow time for water to vaporize and flow out of the reactor core. A long prompt neutron lifetime is of obvious advantage.

For self-regulation it is necessary that the void coefficient α be negative. For given values of k^0_{eff} and l, the greater the magnitude of the negative α, the lower the total heat release required to shut down the reactor.

In this study the prompt neutron lifetime and void coefficient have been calculated for research reactors using M.T.R. (Materials Testing Reactor)-type fuel elements with various combinations of light- and heavy-water coolant moderator and reflectors of light water, heavy water, beryllium, and graphite.

CALCULATION OF NEUTRON LIFETIME AND VOID COEFFICIENT

An equation for calculating mean thermal lifetime of prompt neutrons is derived in the Appendix:

$$l = \frac{\int_V \frac{\phi_s{}^*\phi_s}{v_s}\, dV}{\int_V \frac{k_\infty}{p}\, \Sigma_s \phi_f{}^* \phi_s\, dV} \qquad (3)$$

where the volume integrals are taken over the entire reactor volume. The subscripts s and f refer to properties of neutrons in the thermal and fast groups respectively. The asterisk refers to adjoint properties. Other terms are defined under notation. The numerator of Equation (3) gives the statistically weighted population of thermal neutrons in the reactor, which, for a thermal reactor, is essentially equal to the total neutron population. The denominator gives the statistically weighted neutron production rate for the reactor. The ratio of the two is the time to replace one neutron population by another generation of neutrons.

The equation for the fractional void coefficient $[\partial k_{eff}/k_{eff}]/[\partial V_v/V_{mod}]$, or the differential change in effective reproduction factor k_{eff} per differential fraction of moderator converted to steam, is derived in the Appendix.

If one considers the effect of a differential volume ∂V_v of steam formation on the effective reproduction factor, the steam void coefficient α is given by

$$\alpha = \frac{1}{V_{mod}} \frac{\partial k_{eff}/k_{eff}}{\partial V_v/V_{mod}} \qquad (4)$$

CALCULATION OF CRITICAL MASS AND FLUX DISTRIBUTION

For simplicity the calculations were made for spherical reactor cores completely surrounded by reflector. The radius of the light-water cores was determined as that radius required for criticality when the core contains approximately the same concentration of water, uranium, and aluminum as the borax reactor and is surrounded by an infinitely thick reflector of light water. When other reflectors were considered, the core radius and aluminum and water concentration were held constant, but the amount of uranium 235 alloyed in the aluminum fuel elements was varied to obtain criticality. In every case it was assumed that the reactor core contained 5% excess absorption in the form of removable control elements, to allow for operational requirements of reactivity.

In a heavy-water core M.T.R.-type fuel elements are spaced several inches apart and there is considerable latitude in the core size; so various core radii as well as reflector combinations were considered in this study.

Also, heavy-water cores require lower critical masses and fewer fuel elements than do light-water cores and the aluminum content was therefore reduced to 57.8 kg. for the heavy-water cores. Five per cent excess absorption was assumed.

Nuclear data used in these calculations are shown in Table 1. All calculations were made by means of two-group diffusion theory, and the core was treated as a homogeneous medium.

RESULTS AND DISCUSSION

Light-water Cores

Results for the light-water cores are shown in Table 2. Figure 1 shows flux distributions for the seven light-water cases, normalized to unit thermal flux at the reactor center. Because of the short slowing-down length and diffusion length of light water, the results of case 1 apply as well for an infinitely thick-water reflector. For the borax experiment (*1*) the reactor lifetime was 0.000065 sec. and the fractional void coefficient was −0.24. These are near the values calculated for the water-reflected, water-moderated reactor (case 1).

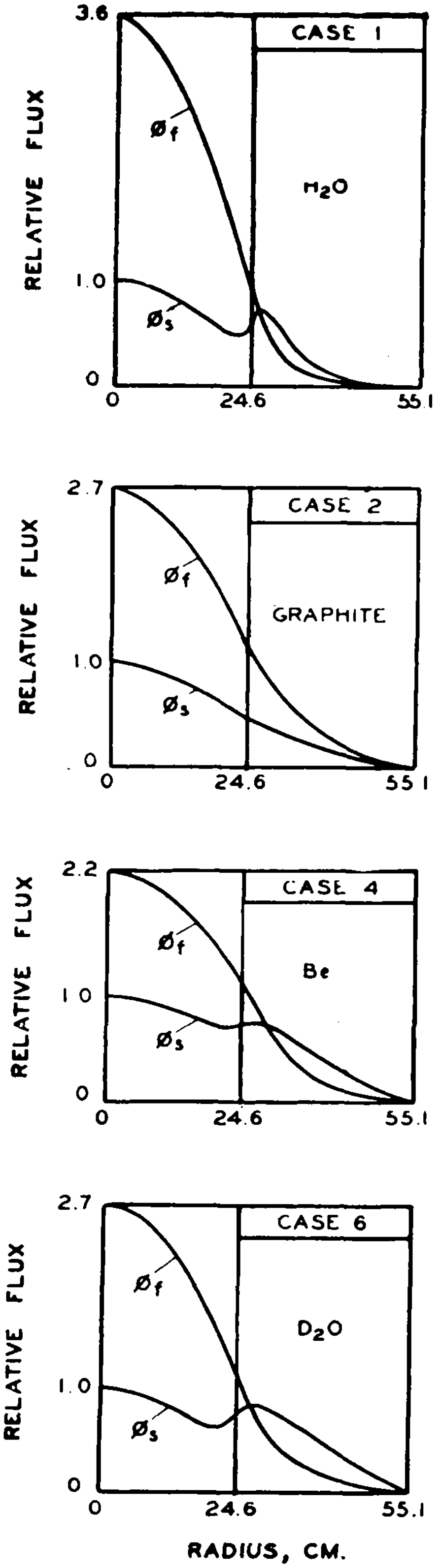
The savings in critical mass as graphite, beryllium, or heavy water replace the light-water reflector are apparent. As would be expected, beryllium is the most effective reflector for fuel savings. The prompt neutron lifetime can be increased considerably over case 1 by use of a better reflector. The lifetime gain with beryllium is due largely to the lower critical mass and hence to the lower value of $k_\infty \Sigma_a$ in the core. With heavy-water reflector the almost fivefold increase in lifetime results mainly from the buildup of a large neutron population in the reflector. This is also evidenced by comparing the flux plots for cases 1 and 7 in Figure 1.

For a given reflector material, increasing the reflector thickness increases the neutron lifetime because of the lower critical mass and the greater neutron population. Reactors reflected by graphite and heavy water benefit the most from the additional 30.5 cm. of reflector because of the large slowing-down length in graphite and the large thermal diffusion length in heavy water. Large values of these properties result in appreciable neutron concentration at considerable distances from the core, and additional reflector thickness is effective in allowing neutron population buildup and in reflecting neutrons back to the core.

The gains in lifetime with better reflectors are counterbalanced by the less favorable void coefficients. In fact, the positive void coefficients in cases 4 and 5 mean that these reactors are definitely unstable to density changes. The negative void coefficients in cases 3 and 7 are so near zero as to be of little value in providing a means of self-shutdown of the reactor.

The less favorable and positive void coefficients result from the fact that a good reflector reduces the neutron leakage. Any loss in core moderation and increased core leakage from void formation is of lesser importance in affecting the neutron balance than is a poorly reflected or bare reactor. The main result of voiding light-water moderator, which has high neutron absorption, is to decrease parasitic capture, and this tends to make the reactor supercritical. The well-reflected reactor is particularly sensitive to changes in parasitic capture of neutrons by light-water moderator because the reactor has a relatively low critical mass and a much larger fraction of thermal neutrons are being absorbed by water than with a poorly reflected reactor of the same core size.

These results must be qualified, however, as practical research reactors using M.T.R.-type fuel elements have parallelopiped rather than spherical cores, and it is



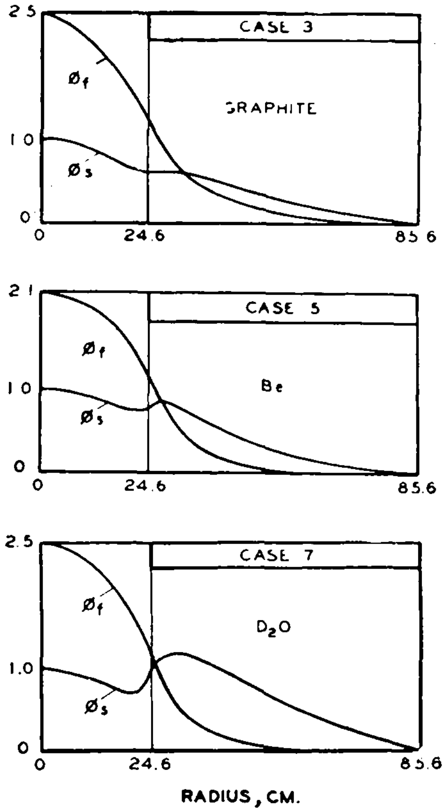


Fig. 1. Flux distributions for light-water cores.

usually necessary to use the water coolant as the reflector on the two core faces through which the water flows. If the other four core faces are surrounded by a better reflector than light water, the trends will be toward more favorable lifetimes and less favorable void coefficients than in the borax case, but the differences in these properties from the borax case will not be so great as those listed in Table 2. For example, the Materials Testing Reactor at Arco, Idaho, is a light-water core reflected on two opposing faces by light-water coolant and on the remaining faces by approximately 30 cm. of beryllium and 120 cm. of graphite. It is reported that the void coefficient for this reactor is negative.

TABLE 1. NUCLEAR DATA USED IN REACTOR CALCULATIONS

Nuclide	Density, g./cc.	Microscopic cross sections, barns: Thermal absorption*	Microscopic cross sections, barns: Thermal transport	Fermi age, sq. cm.
U^{235}	....	597		
H_2O	1.0	0.585	62.9 (3)	31.4
$D_2O(0.25\% H_2O)$	1.1	0.00229	12.0	126.0
Be	1.85	0.00865	5.65 (3)	98.0
C	1.60	0.00412	4.24 (3)	364
Al	2.7	0.204	1.32 (2)	

$$\alpha \text{ for } U^{235} = \frac{\text{capture cross section}}{\text{fission cross section}} = 0.184$$

For the light-water core:

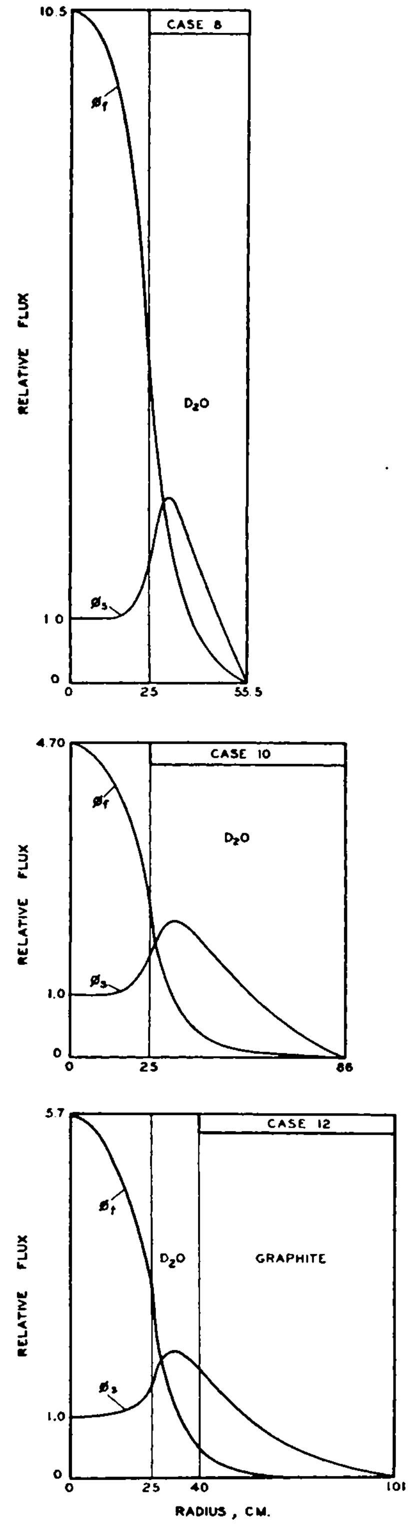
Aluminum-to-water volume ratio = 0.626
Fermi age = 59.1 sq. cm. (4)
Fast-diffusion coefficient = 1.242 cm.

For Al-D_2O mixtures at 20°C. where $X = \frac{\text{aluminum volume}}{\text{water volume}}$

$$\tau = 126.0 + 190.0X \text{ sq. cm.}$$

$$D_f = 1.28 + 0.602X - 0.125X^2 \text{ cm.}$$

*Thermal absorption cross sections from BNL-325 (1), averaged for a Maxwell-Boltzmann distribution of neutrons at 20°C.



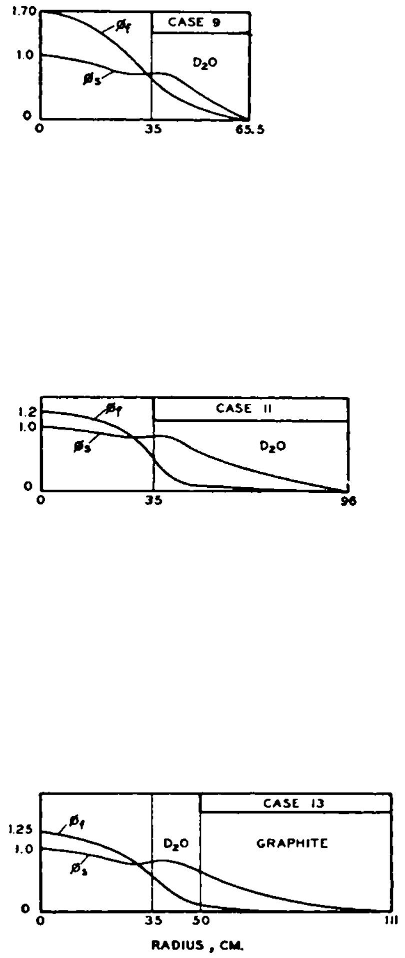


Fig. 2. Flux distributions for heavy-water cores.

Heavy-water Cores

Calculated values of critical mass, neutron lifetime, and void coefficients for the heavy-water cores with various reflectors are presented in Table 3, and flux plots are shown in Figures 2 and 3. The most striking comparison between these results and those of Table 2 lies in the much larger values of neutron lifetimes for heavy-water cores. The lifetimes of all heavy-water cores other than case 9 are much greater even than the heavy-water–reflected light-water cores (cases 6 and 7) of Table 2 because the lower macroscopic thermal absorption cross section in a heavy-water core allows a core neutron to live considerably longer before dying by absorption.

The steam void coefficients in Table 3 are in some cases less favorable and in some cases more favorable than the coefficient of the borax-type reactor (case 1), but they are all more negative than for the other light-water cores of Table 2. As heavy water is such a weak absorber of neutrons, loss of parasitic absorption through steam formation in heavy water in the core contributes little to reactivity changes. Decreased moderation and increased fast leakage as voids are formed account for the negative void coefficients for the heavy-water cores.

Increasing the core radius at constant reflector thickness results in a more negative fractional void coefficient for heavy-water cores. This results from the greater dependence of the smaller core (25-cm. radius) upon the reflector as a source of thermal neutrons. This can be seen from the flux plots, Figures 2 and 3,

TABLE 2. NUCLEAR PROPERTIES OF CORES COOLED AND MODERATED BY LIGHT WATER

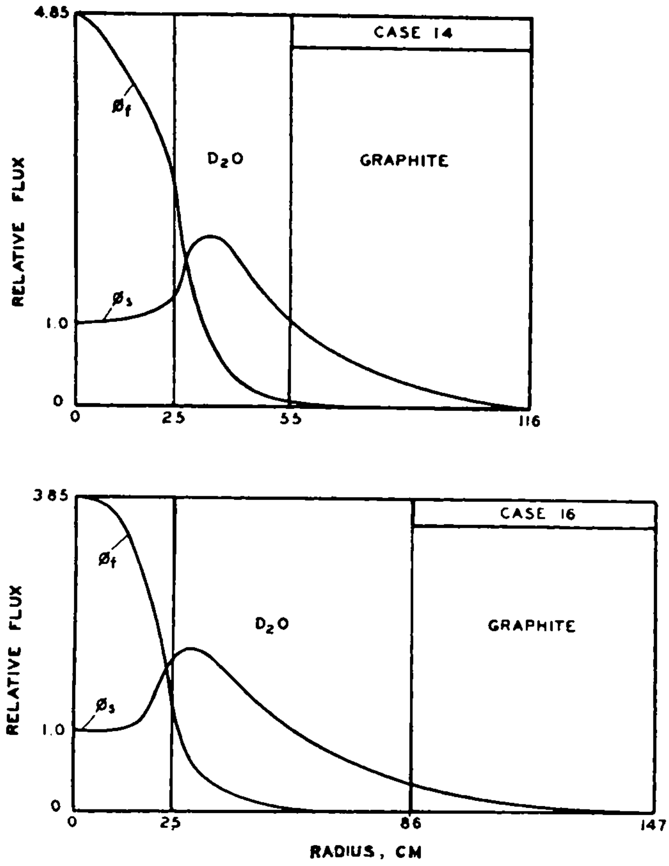
Core radius: 24.6 cm.
Fuel elements: M.T.R. plate type containing 64.7 kg. of aluminum and amounts of U^{235} indicated below.

Case	1	2	3	4	5	6	7
Reflector	H_2O	C	C	Be	Be	D_2O	D_2O
Reflector thickness, cm.	30.5	30.5	61	30.5	61	30.5	61
k_∞	1.594	1.438	1.386	1.305	1.290	1.457	1.391
Critical mass of U^{235}, kg.	2.257	1.538	1.371	1.157	1.121	1.608	1.387
Prompt neutron lifetime, 10^{-3} sec.	0.0629	0.109	0.202	0.161	0.207	0.144	0.329
Fractional void coefficient $(\partial k_{eff}/k_{eff})/(\partial V_v/V_{H_2O})$	−0.277	−0.110	−0.0391	+0.0536	+0.0735	−0.102	−0.0130
Steam void coefficient $(\partial k_{eff}/k_{eff})/\partial V_v$, 10^{-6} cm.$^{-3}$	−7.21	−2.86	−1.02	+1.40	+1.91	−2.66	−0.338

which show a significant peak of thermal flux in the reflector for the 25-cm. cores and a positive gradient in thermal flux over a large portion of the core volume. As the smaller core is voided, a significant fraction of the increased fast leakage is moderated in the reflector and returned to the core. Since most of the thermal neutrons diffuse toward the center of the small core, loss of heavy water allows them to penetrate nearer the center of the core, where their statistical weight is greatest.*

The 35-cm. heavy-water cores depend much less upon the reflector for moderation, as evidenced by the lower reflector flux peaks in Figure 2. Likewise, thermal neutrons diffuse outward over most of the 35-cm. cores, and with loss of heavy water thermal neutrons tend to penetrate farther away from the regions of high statistical weight in the center of the core. The over-all effect is greater negative fractional void coefficients for the larger heavy-water core because of effectively greater loss of neutrons by leakage when voids are formed in the core moderator.

*The adjoint fluxes, which determine statistical weights, are greatest in the center and decrease monotonically with increasing core radius.



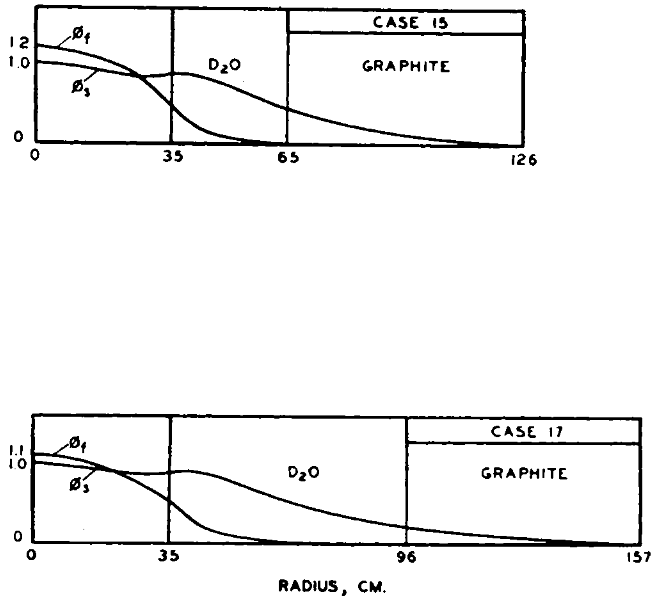


Fig. 3. Flux distributions for heavy-water cores.

TABLE 3. NUCLEAR PROPERTIES OF CORES COOLED AND MODERATED BY HEAVY WATER

Fuel elements: 57.8 kg. of aluminum containing amount of U^{235} indicated below

Case	8	9	10	11	12	13	14	15	16	17
Core radius, cm.	25	35	25	35	25	35	25	35	25	35
Thickness of D_2O reflector, cm.	30.5	30.5	61	61	15	15	30.5	30.5	61	61
Thickness of graphite reflector, cm.	0	0	0	0	61	61	61	61	61	61
k_∞	1.854	1.734	1.732	1.645	1.746	1.658	1.721	1.637	1.686	1.608
Critical mass of U^{235}, kg.	2.612	1.281	1.227	0.890	1.315	0.933	1.166	0.862	1.008	0.782
Prompt neutron mean lifetime, 10^{-3} sec.	0.239	0.444	0.708	0.955	0.633	0.872	0.835	1.087	1.223	1.490
Fractional void coefficient $(\partial k_{eff}/k_{eff})/(\partial V_v/V_{D_2O})$	−0.272	−0.330	−0.191	−0.232	−0.200	−0.242	−0.184	−0.223	−0.166	−0.200
Steam void coefficient $(\partial k_{eff}/k_{eff})/\partial V_v$, 10^{-6} cm.$^{-3}$	−6.19	−2.08	−4.34	−1.466	−4.55	−1.53	−4.18	−1.41	−3.78	−1.26

Obviously this effect must eventually be reversed as core size is increased further. For an infinitely large reactor there is no leakage, and, if no resonance absorbers are present, loss of moderator by voiding can only increase reactivity due to loss of poisons.

The reactivity change per unit volume of steam formed is greater for the smaller cores, because a unit volume of steam is a much greater fraction of the total moderator volume for the smaller cores. The reactivity change per unit volume of steam is probably more significant than the fractional void coefficient in assessing the inherent safety of a reactor, as it is a measure of the reactivity change per unit amount of heat liberated.

The results of Table 3 for spherically reflected cores should be good approximations for a practical parallelopiped reactor core, because the core, cooled, moderated, and reflected by heavy water, will be surrounded on all sides by heavy water. The graphite outer reflector may be missing beyond two of the core faces, but corrections for this are of less importance.

CONCLUSIONS

It is clear that the greatest safety gain over the borax reactor is due to the use of heavy water for both moderator and reflector. This is shown by the large increase in prompt neutron lifetime with only slight decrease in reactivity change per unit volume of steam.

It is difficult to give a precise measure of the gain in safety since it is not possible to determine quantitatively the amounts of reactivity which, if suddenly imposed on a given reactor, would produce the same result as the borax experiments. Analytical approaches to this have not been completely successful because of lack of information concerning transient heat transfer to boiling water at high heat fluxes and in narrow channels and insufficient knowledge of characteristics of two-phase fluid flow under rapidly varying conditions of pressure and steam quality.

ACKNOWLEDGMENT

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NOTATION

A, C, F, G = coefficients of real fluxes in core and reflector
A^*, C^*, F^*, G^* = coefficients of adjoint fluxes in core and reflector
D = diffusion coefficient, cm.
k_∞ = infinite reproduction factor
k_{eff} = effective reproduction factor
k^0_{eff} = effective reproduction factor immediately after excess reactivity is introduced
l = mean lifetime of prompt neutrons, sec.
L = diffusion length, cm.
p = resonance escape probability
r = radius, cm.
R_1 = radius of reactor core, cm.
R_2 = outside radius of reflector, cm.
S_1, S_2, S_3 = coupling coefficients between fast and thermal fluxes
S_1^*, S_2^*, S_4^* = coupling coefficients between fast and thermal adjoint fluxes
t = time, sec.
T = reactor period after transients from higher flux harmonics have died out, sec.
v = neutron speed, cm./sec.
V = volume, cc.
W, X, Y, Z = space-dependent functions in the expressions for neutron flux
α = steam void coefficient, change in reactivity per unit volume of steam formed, cm.$^{-3}$
η = fission neutrons per neutron absorbed in fuel
κ = reciprocal diffusion length, cm.$^{-1}$
λ = reciprocal period, sec.$^{-1}$
μ = positive component of reciprocal buckling, cm.$^{-1}$
ν = negative component of reciprocal buckling, cm.$^{-1}$
ϕ = neutron flux, neutrons/(sq. cm.)(sec.)
ϕ' = spatial component of neutron flux after a perturbation, neutrons/(sq. cm.)(sec.)
ϕ'' = spatial component of neutron flux after a uniformly occurring perturbation in k_∞, neutrons/(sq. cm.)(sec.)
ϕ^* = adjoint neutron flux, neutrons/(sq. cm.)(sec.)
Σ = macroscopic cross section, cm.$^{-1}$
τ = Fermi age, sq. cm.

Subscripts

c = reactor core
f = properties of fast-neutron group
mod = moderator in reactor core
n = eigenvalue of space-dependent solution of neutron flux
o = first eigenvalue of space-dependent solution of neutron flux
s = properties of thermal neutrons
tr = transport properties
v = steam voids

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APPENDIX

Derivation of Equations for Void Coefficients and Neutron Lifetimes

The time-dependent neutron-balance equations for a two-group model of a thermal reactor are

$$(\nabla\cdot D_s\nabla - \Sigma_s)\phi_s + p\Sigma_f\phi_f = \frac{1}{v_s}\frac{\partial\phi_s}{\partial t} \quad \text{(A-1)}$$

$$\frac{k_\infty\Sigma_s}{p}\phi_s + (\nabla\cdot D_f\nabla - \Sigma_f)\phi_f = \frac{1}{v_f}\frac{\partial\phi_f}{\partial t} \quad \text{(A-2)}$$

where Σ_f refers to the slowing-down cross section for the fast group. Resonance absorption is assumed to occur in a narrow energy band between the thermal and fast group. Effects of delayed neutrons have been ignored.

The solutions of Equations (A-1) and (A-2) are of the form

$$\phi_s(r, t) = \sum A_n\phi_{sn}(\mathbf{r})e^{\lambda_n t} \quad \text{(A-3)}$$

$$\phi_f(r, t) = \sum A_n\phi_{fn}(\mathbf{r})e^{\lambda_n t} \quad \text{(A-4)}$$

where the functions ϕ_{sn} and ϕ_{fn} satisfy the time-independent equations

$$(\nabla\cdot D_s\nabla - \Sigma_s - \lambda_n/v_s)\phi_{sn} + p\Sigma_f\phi_{fn} = 0 \quad \text{(A-5)}$$

$$\frac{k_\infty}{p}\Sigma_s\phi_{sn} + (\nabla\cdot D_f\nabla - \Sigma_f - \lambda_n/v_f)\phi_{fn} = 0 \quad \text{(A-6)}$$

The adjoint fluxes ϕ_{sn}^* and ϕ_{fn}^* are those which are the solutions of the differential equations resulting from transposing the matrix of coefficients of Equations (A-5) and (A-6).

$$(\nabla\cdot D_s\nabla - \Sigma_s - \lambda_n/v_s)\phi_{sn}^* + \frac{k_\infty}{p}\Sigma_s\phi_{fn}^* = 0 \quad \text{(A-7)}$$

$$p\Sigma_f\phi_{sn}^* + (\nabla\cdot D_f\nabla - \Sigma_f - \lambda_n/v_f)\phi_{fn}^* = 0 \quad \text{(A-8)}$$

A just critical reactor ($\lambda = 0$) will be considered in which the following small changes are made:

$$D_s \to D_s + \delta D_s \qquad D_f \to D_f + \delta D_f$$
$$\Sigma_s \to \Sigma_s + \delta\Sigma_s \qquad \Sigma_f \to \Sigma_f + \delta\Sigma_f$$
$$\frac{k_\infty\Sigma_s}{p} \to \frac{k_\infty\Sigma_s}{p} + \delta\left(\frac{k_\infty\Sigma_s}{p}\right) \qquad p\Sigma_f \to p\Sigma_f + \delta(p\Sigma_f) \quad \text{(A-9)}$$

After the changes summarized by Equation (A-9), the reactor flux will fall or rise with an ultimate steady period $1/\lambda$. The space-dependent part of the perturbed fluxes ϕ_{so}' and ϕ_{fo}' obey the equations

$$[\nabla\cdot(D_s + \delta D_s)\nabla - (\Sigma_s + \delta\Sigma_s) - \lambda/v_s]\phi_{so}' + [p\Sigma_f + \delta(p\Sigma_f)]\phi_{fo}' = 0 \quad \text{(A-10)}$$

$$\left[\frac{k_\infty\Sigma_s}{p} + \delta\left(\frac{k_\infty\Sigma_s}{p}\right)\right]\phi_{so}' + \left[\nabla\cdot(D_f + \delta D_f)\nabla - (\Sigma_f + \delta\Sigma_f) - \frac{\lambda}{v_f}\right]\phi_{fo}' = 0 \quad \text{(A-11)}$$

where the subscript o refers to the fundamental mode of the space-dependent solutions.

To compute λ, one multiplies (A-10) by ϕ_{so}^* and (A-11) by ϕ_{fo}^* and adds. λ is set equal to zero in Equations (A-7) and (A-8), (A-7) is multiplied by ϕ_{so}', (A-8) by ϕ_{fo}', and both are added. The second sum is subtracted from the first and the resulting equation is integrated over the reactor volume. The result, arrived at from Green's theorem and the fact that the flux vanishes at the outer surface of the reactor, is

$$\lambda = \frac{\int_V \left[\delta(p\Sigma_f)\phi_{so}^*\phi_{fo}' + \delta\left(\frac{k\Sigma_s}{p}\right)\phi_{fo}^*\phi_{so}' - \delta\Sigma_s\phi_{so}^*\phi_{so}' - \delta\Sigma_f\phi_{fo}^*\phi_{fo}' - \nabla\phi_{so}^*\cdot\delta D_s\nabla\phi_{so}' - \nabla\phi_{fo}^*\cdot\delta D_f\nabla\phi_{fo}'\right] dV}{\int_V \left[\frac{\phi_{so}^*\phi_{so}'}{v_s} + \frac{\phi_{fo}^*\phi_{fo}'}{v_f}\right] dV} \quad \text{(A-12)}$$

A reactivity change is described by that equivalent to a uniform change in k_∞, $\overline{\delta k}_\infty$, which if imposed on the just critical reactor would lead to the same period, $1/\lambda$, as do all the actual changes. The neutron flux ϕ_o'' in this equivalent reactor will satisfy

$$(\nabla\cdot D_s\nabla - \Sigma_s - \lambda/v_s)\phi_{so}'' + p\Sigma_f\phi_{fo}'' = 0 \quad \text{(A-13)}$$

$$\frac{\Sigma_s}{p}(k_\infty + \overline{\delta k}_\infty)\phi_{so}'' + (\nabla\cdot D_f\nabla - \Sigma_f - \lambda/v_f)\phi_{fo}'' = 0 \quad \text{(A-14)}$$

By the same technique that led to (A-12), the relation between λ and $\overline{\delta k}_\infty$ is found to be

$$\lambda = \frac{\overline{\delta k}_\infty \int_V \frac{\Sigma_s}{p}\phi_{fo}^*\phi_{so}''\, dV}{\int_V \left(\frac{\phi_{so}^*\phi_{so}''}{v_s} + \frac{\phi_{fo}^*\phi_{fo}''}{v_f}\right) dV} \quad \text{(A-15)}$$

For a single multiplying region the fractional change $\overline{\delta k}_\infty/k_\infty$ is identical with the fractional change in the effective reproduction factor $\overline{\delta k_{eff}/k_{eff}}$. One may solve explicitly for $\overline{\delta k}_\infty/k_\infty$ by eliminating λ from (A-12) and (A-15) and making the approximation of first-order perturbation theory that

$$\phi_0'' = \phi_0' = \phi$$

where ϕ is the solution of Equations (A-5) and (A-6) with λ equal to zero.

There results

$$\frac{\overline{\delta k}_\infty}{k_\infty} = \frac{\int_V \left[\delta(p\Sigma_f)\phi_s^*\phi_f + \delta\left(\frac{k_\infty\Sigma_s}{p}\right)\phi_f^*\phi_s - \delta\Sigma_s\phi_s^*\phi_s - \delta\Sigma_f\phi_f^*\phi_f - \nabla\phi_s^*\cdot\delta D_s\nabla\phi_s - \nabla\phi_f^*\cdot\delta D_f\nabla\phi_f\right] dV}{\int_V \frac{k_\infty\Sigma_s}{p}\phi_f^*\phi_s\, dV} \quad \text{(A-16)}$$

As delayed neutrons in the original equations have been neglected, the reciprocal asymptotic period λ, when all harmonics have died out, is related to the effective reproduction factor k_{eff} and mean lifetime of prompt neutrons l by

$$\lambda = \frac{k_{eff} - 1}{l} \quad \text{(A-17)}$$

For perturbations to an initially critical reactor, k_{eff} initially equals unity, and so for a reactor with a single multiplying region

$$k_{eff} - 1 = \frac{\delta k_{eff}}{k_{eff}} = \frac{\overline{\delta k}_\infty}{k_\infty} \quad \text{(A-18)}$$

and

$$\lambda = \frac{\overline{\delta k}_\infty}{k_\infty l} \quad \text{(A-19)}$$

Solving for l from Equations (A-15) and (A-19) one obtains

$$l = \frac{\int_V \left[\frac{\phi_s^*\phi_s}{v_s} + \frac{\phi_f^*\phi_f}{v_f}\right] dV}{\int_V \frac{k_\infty\Sigma_s}{p}\phi_f^*\phi_s\, dV} \quad \text{(A-20)}$$

For thermal reactors the second term in the numerator of the equation is much smaller than the first term and may be neglected, so that

$$l \cong \frac{\int_V \frac{\phi_s^*\phi_s}{v_s}\, dV}{\int_V \frac{k_\infty\Sigma_s}{p}\phi_f^*\phi_s\, dV} \quad \text{(A-21)}$$

This is equivalent to saying that the slowing-down time is small compared with the thermal lifetime.

A spherical reactor core of radius R_1 surrounded by a spherical reflector shell of outside radius R_2 will be considered. The reactor is assumed to be just critical before perturbations occur. The solutions of the real and adjoint fluxes in the core and reflector are as follows:

Core

$$\phi_{fc} = AW + CX \quad \text{(A-22)}$$

$$\phi_{sc} = AS_1W + CS_2X \quad \text{(A-23)}$$

where

$$W = \frac{\sin \mu r}{r} \quad \text{(A-24)}$$

$$X = \frac{\sinh \nu r}{r} \quad \text{(A-25)}$$

$$S_1 = \frac{D_{fc}}{D_{sc}\tau_c}\left[\frac{p}{\kappa_{sc}^2 + \mu^2}\right] \quad \text{(A-26)}$$

$$S_2 = \frac{D_{fc}}{D_{sc}\tau_c}\left[\frac{p}{\kappa_{sc}^2 - \nu^2}\right] \quad \text{(A-27)}$$

$$\mu^2 = \frac{1}{2}\left[-\left(\frac{1}{\tau_c} + \frac{1}{L_{sc}^2}\right) + \sqrt{\left(\frac{1}{\tau_c} + \frac{1}{L_{sc}^2}\right)^2 + \frac{4(k_\infty - 1)}{\tau_c L_{sc}^2}}\right] \quad \text{(A-28)}$$

$$\nu^2 = -\frac{1}{2}\left[-\left(\frac{1}{\tau_c} + \frac{1}{L_{sc}^2}\right) - \sqrt{\left(\frac{1}{\tau_c} + \frac{1}{L_{sc}^2}\right)^2 + \frac{4(k_\infty - 1)}{\tau_c L_{sc}^2}}\right] \quad \text{(A-29)}$$

$$\kappa_{sc}^2 = \frac{1}{L_{sc}^2} \quad \text{(A-30)}$$

$$\phi_{fc}^* = A^*W + C^*X \quad \text{(A-31)}$$

$$\phi_{sc}^* = A^*S_1^*W + C^*S_2^*X \quad \text{(A-32)}$$

$$S_1^* = \frac{\mu^2\tau + 1}{p} \quad \text{(A-33}a\text{)}$$

$$S_2^* = \frac{1 - \nu^2\tau}{p} \quad \text{(A-33}b\text{)}$$

Reflector

$$\phi_{fr} = FY \quad \text{(A-34)}$$

$$\phi_{sr} = FS_3Y + GZ \quad \text{(A-35)}$$

where

$$Y = \frac{\sinh \kappa_{fr}(R_2 - r)}{r} \quad \text{(A-36)}$$

$$Z = \frac{\sinh \kappa_{sr}(R_2 - r)}{r} \quad \text{(A-37)}$$

$$\kappa_{fr}^2 = \frac{1}{\tau_r} \quad \text{(A-38)}$$

$$S_3 = \frac{D_{fr}}{D_{sr}}\left(\frac{1}{\kappa_{sr}^2\tau_r - 1}\right) \quad \text{(A-39)}$$

$$\phi_{fr}^* = F^*Y + G^*Z \quad \text{(A-40)}$$

$$\phi_{sr}^* = G^*S_4^*Z \quad \text{(A-41)}$$

where

$$S_4^* = 1 - \kappa_{sr}^2\tau_r \quad \text{(A-42)}$$

The fast and thermal fluxes, real and adjoint, must satisfy the boundary conditions of continuity of flux and neutron current at the core-reflector interface (at R_1):

$$\left.\begin{array}{lllll} WA & + XC & - YF & & = 0 \\ D_{fc}\nabla WA & + D_{fc}\nabla XC & - D_{fr}\nabla YF & & = 0 \\ S_1WA & + S_2XC & - S_3YF & - ZG & = 0 \\ D_{sc}S_1\nabla WA & + D_{sc}S_2\nabla XC & - D_{sr}S_3\nabla YF & - D_{sr}\nabla ZG & = 0 \end{array}\right\} \quad \text{(A-43)}$$

$$\left.\begin{array}{lllll} WA^* & + XC^* & - YF^* & - ZG^* & = 0 \\ D_{fc}\nabla WA^* & + D_{fc}\nabla XC^* & - D_{fr}\nabla YF^* & - D_{fr}\nabla ZG^* & = 0 \\ S_1^*WA & + S_2^*XC^* & & - S_4^*ZG^* & = 0 \\ D_{sc}S_1^*\nabla WA^* & + D_{sc}S_2^*\nabla XC^* & & - D_{sr}S_4^*\nabla ZG^* & = 0 \end{array}\right\} \quad \text{(A-44)}$$

For given R_1 and R_2, the critical fuel concentration (or k_∞) can be calculated from (A-43). Relative values of the coefficients A, C, F, and G can then be calculated for (A-43), and relative values of A^*, C^*, F^*, and G^* from (A-44).

If there are two reflector regions, as with the D_2O-core reactors, the flux expressions for the inner reflector must contain both sinh and cosh terms, and an additional set of boundary conditions must be written for the interface between the two reflectors. The treatment is otherwise the same as for the single reflector.

Flux expressions obtained as above can be substituted in (A-21) to determine the prompt neutron lifetime.

Void Coefficient

When a differential volume ∂V_v of steam is formed, the total moderator volume increases by the same amount ($\partial V_{mod} = \partial V_v$), and a volume ∂V_{mod} is displaced from the reactor core. The fractional change in k_{eff} per fractional increase in core moderator volume is obtained from (A-16):

$$\frac{\partial k_{eff}/k_{eff}}{\partial V_v/V_{mod}} = \frac{\overline{\partial k_\infty/k_\infty}}{\partial V_{mod}/V_{mod}} = \frac{1}{\int_V \frac{k_\infty\Sigma_s}{p}\phi_f^*\phi_s\,dV} \cdot \left[\frac{\partial(p\Sigma_f)_c}{\partial V_{mod}/V_{mod}}\int_{V_c}\phi_s^*\phi_f\,dV + \frac{\partial\left(\frac{k_\infty\Sigma_s}{p}\right)}{\partial V_{mod}/V_{mod}}\int_{V_c}\phi_f^*\phi_s\,dV - \frac{\partial\Sigma_{fc}}{\partial V_{mod}/V_{mod}}\int_{V_c}\phi_f^*\phi_f\,dV - \frac{\partial\Sigma_{sc}}{\partial V_{mod}/V_{mod}}\int_{V_c}\phi_s^*\phi_s\,dV - \frac{\partial D_{sc}}{\partial V_{mod}/V_{mod}}\int_{V_c}\nabla\phi_s^*\cdot\nabla\phi_s\,dV - \frac{\partial D_{fc}}{\partial V_{mod}/V_{mod}}\int_{V_c}\nabla\phi_f^*\cdot\nabla\phi_f\,dV\right] \quad \text{(A-45)}$$

For reactors fueled with fully enriched U^{235}, p is unity and the second term in the brackets of (A-45) drops out.

The differential perturbations are evaluated as follows for a core consisting of U^{235}, aluminum, and moderator:

$$\Sigma_s = \Sigma_s(\text{U}) + \Sigma_s(\text{Al}) + \Sigma_s(\text{mod})$$

As voids are formed, the total volume occupied by moderator atoms increases, and the macroscopic cross section for moderator absorption is given by

$$\Sigma_s'(\text{mod}) = \Sigma_s(\text{mod})\frac{V_{mod}}{V_{mod}'}$$

where the primed quantities are the values after a change in moderator density. For a differential increase in moderator volume,

$$\frac{\partial\Sigma_s}{\partial V_{mod}/V_{mod}} = -\Sigma_s(\text{mod}) \quad \text{(A-46)}$$

$$D_s' = \frac{1}{3\left[\Sigma_{tr}(\text{Al}) + \Sigma_{tr}(\text{mod})\frac{V_{mod}}{V_{mod}'}\right]}$$

so that

$$\frac{\partial D_s}{\partial V_{mod}/V_{mod}} = \frac{D_{sc}^2}{D_s(\text{mod})} \quad \text{(A-47)}$$

The change in slowing-down cross section resulting from an increase in volume occupied by moderator is written by analogy to Equation (A-46):

$$\frac{\partial\Sigma_f}{\partial V_{mod}/V_{mod}} = -\Sigma_f(\text{mod}) \quad \text{(A-48)}$$

The total slowing-down cross section Σ_f will be essentially that of the moderator $\Sigma_f(\text{mod})$, because of the low logarithmic energy decrement of aluminum relative to that of the moderator.

Since the total slowing-down cross section in two-group theory is

$$\Sigma_f = \frac{D_f}{\tau}$$

Equation (A-48) becomes

$$\frac{\partial\Sigma_f}{\partial V_{mod}/V_{mod}} = -\frac{D_f}{\tau} \quad \text{(A-49)}$$

The perturbation in fast diffusion coefficient is written by analogy to Equation (A-47):

$$\frac{\partial D_f}{\partial V_{mod}/V_{mod}} = \frac{D_f^2}{D_f(\text{mod})} \quad \text{(A-50)}$$

In the calculation of a differential void coefficient for a reactor with no resonance capture, Equations (A-46), (A-47), (A-49), and (A-50) are substituted in (A-45). The flux integrals are easily evaluated from the flux expressions obtained as described above. The volume integrals involving products of flux gradients are easily evaluated from Green's theorem.

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